# UNIT #1, Number Sense:

## Chapter #1, Rational Numbers

What is a Rational Number? - A rational number is a number that can be written as the quotient (measure) of two integers where the divisor (number you are dividing by)is not zero. It can be written as a fraction, mixed fraction, decimal or integer.

So basically any number that can be produced in a question from two integers, decimals or fractions is a rational number.

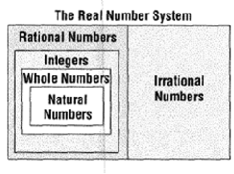
e.g., $3.25, -5.8, \frac{2}{3}, -2, -1\frac{1}{4}$

Irrational numbers are numbers that do not make sense! They are never ending, never repeating decimals. They cannot be written in fraction form. Some examples are the square root of 2 and pi (3.14159...)

A **rational number** is a number that can be written as a simple fraction (ie as a **ratio**).

Example **1.5** is a rational number because 1.5 = **3/2** (it can be written as a fraction)

Here are some more examples:




| Number | As a Fraction | Rational? |
|---|---|---|
| 5 | 5/1 | Yes |
| 1.75 | 7/4 | Yes |
| .001 | 1/1000 | Yes |
| 0.111... | 1/9 | Yes |
| √2 (square root of 2) | ? | NO ! |

Natural Numbers are the positive integers, there has been a debate on whether zero should be included in these numbers. e.g. 0, 1, 2, 3, 4, 5 .........

### In Summary

**Key Idea**

- Rational numbers include integers, fractions, their decimal equivalents, and their opposites.

**Need to Know**

- Rational numbers can be positive, negative, or zero.
- Every integer is a rational number because it can be written as the quotient of two integers.

  For example, some ways $-5$ can be written are $\frac{-5}{1}$, $\frac{5}{-1}$, and $\frac{-10}{2}$.
- Just as with fractions, there are many ways to write the same rational number.

  For example, $-2.5 = \frac{-5}{2} = -\frac{5}{2} = \frac{5}{-2} = -2.50 = -\frac{10}{4}$.
- Every rational number (except 0) has an opposite. For example, $-2\frac{3}{4}$ and $2\frac{3}{4}$ are opposites, since they are both the same distance from 0 on a number line.

### In Summary

**Key Ideas**

- Rationals can be compared in the same ways as integers and fractions.
- Negatives are always less than positives. A negative farther from 0 is always less than a negative closer to 0.

**Need to Know**

- To compare rationals in fraction form, it helps to use mixed number representations, equivalent fractions with common denominators or common numerators, or benchmarks. For example,

  $-3\frac{1}{2} > -4\frac{1}{3}$ since $-3 > -4$

  $-\frac{3}{5} > -\frac{7}{8}$ since $-\frac{24}{40} > -\frac{35}{40}$ or since $-\frac{21}{35} > -\frac{21}{24}$

  $-\frac{2}{3} < -\frac{1}{5}$ since $-\frac{2}{3} < -\frac{1}{2}$ and $-\frac{1}{5} > -\frac{1}{2}$
- You can also express all the rational numbers as decimals and compare them in decimal form.

## Adding and subtracting fractions

- Adding and subtracting fractions are very similar, follow the steps below, the only difference is that once you have common denominators, add or subtract depending on the symbol.

- **These are the steps for adding and subtracting fractions!!!**

1. **Find equivalent fractions for the fractions you are adding or subtracting**
2. **Make sure that they have common denominators!!**
3. **Add the numerators together, leaving the denominator the same**
4. **Put** **lowest t**

e.g. $\frac{2}{3} + \frac{1}{4} = \frac{\square}{\square}$

$\frac{2}{3} \xrightarrow{\times 4} \frac{8}{12}$ $\quad$ $\frac{1}{4} \xrightarrow{\times 3} \frac{3}{12}$

$\frac{8}{12} + \frac{3}{12} = \frac{11}{12}$

$\frac{3}{4} - \frac{1}{7} = \frac{\square}{\square}$

$\frac{3}{4} \xrightarrow{\times 7} \frac{21}{28}$ $\quad$ $\frac{1}{7} \xrightarrow{\times 4} \frac{4}{28}$

$\frac{21}{28} - \frac{4}{28} = \frac{17}{28}$

## Adding and subtracting mixed numbers

- Before you can add or subtract mixed numbers, (mixed fractions) you must put them into the right form.

- Remember to choose your strategy so that you are not confused. For example if you are subtracting mixed numbers and the fraction part of the second number is less than the first, you might be left with a negative fraction when you're done! In this situation it would be simpler to use the improper fraction method.

Mixed Number Strategy - add or subtract the whole numbers separately, then do the same with the fractions, put them back together for a final answer. Remember to leave your answer in lowest form!

$5\frac{4}{5} + 2\frac{2}{3}$

$5 + 2 = 7$

$\frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} = 1\frac{7}{15} + 7 = \boxed{8\frac{7}{15}}$

Convert this improper fraction to a mixed fraction

Improper Fraction Strategy - convert all fractions you are adding or subtracting into improper fractions first, them find common denominators and solve, remember to leave your answer in lowest form!

$5\frac{4}{5} + 2\frac{2}{3}$

$\frac{5}{5}+\frac{5}{5}+\frac{5}{5}+\frac{5}{5}+\frac{5}{5}+\frac{4}{5}$ $\quad$ $\frac{3}{3}+\frac{3}{3}+\frac{2}{3}$

$\frac{29}{5} + \frac{8}{3} = \frac{87}{15} + \frac{40}{15} = \frac{127}{15} = \boxed{8\frac{7}{15}}$

Convert this improper fraction to a mixed fraction

## 2.1 & 2.3 & 2.5 - Multiplying fractions

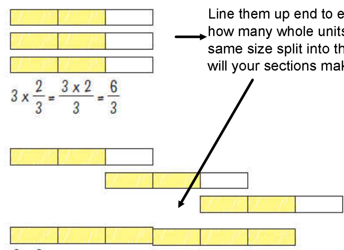
- One whole unit can be represented by the fraction $\frac{1}{1}$ or by any fraction that show all parts of the whole filled e.g. $\frac{2}{2}$ or $\frac{4}{4}$ Most times though it is easiest to use $\frac{1}{1}$
- So if you have four whole units (4) then the fraction would be $\frac{4}{1}$

- When Multiplying whole numbers, fractions or mixed numbers the safest pattern to follow is to convert all terms to proper or improper fractions. At that point you multiply all numerators together and multiply all the denominators.

Step #1 - Convert terms to improper or proper fractions
Step #2 - Multiply the numerators
Step #3 - Multiply the denominators
Step #4 - Put your answer into lowest terms or a mixed fraction

$3 \text{ or } \frac{3}{1} \times \frac{2}{3}$

$3 \times \frac{2}{3}$ is 3 sets of $\frac{2}{3}$.



Line them up end to end, how many whole units the same size split into thirds will your sections make?

$3 \times \frac{2}{3} = \frac{3 \times 2}{3} = \frac{6}{3}$

$\frac{3 \times 2}{3} = 2$

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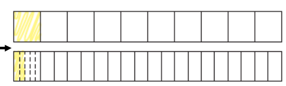
$\frac{2}{5} \times \frac{1}{10}$

Split a portion of the second fraction that equals the first fraction. That amount of the whole unit is the answer!



$\frac{2}{5} \text{ of } \frac{1}{10} = \frac{2}{50}$

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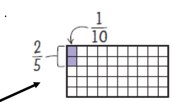
$\frac{2}{5} \times \frac{1}{10}$

Using a grid the horizontal lines make up one fraction, the vertical lines the second fraction. Fill in the portions so that both fractions are used and you get your final answer!




$$\frac{2}{5} \times \frac{1}{10} = \frac{2 \times 1}{5 \times 10} = \frac{2}{50} = \frac{1}{25}$$

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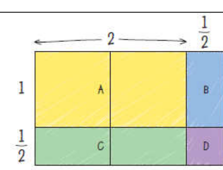
$1\frac{1}{2} \times 2\frac{1}{2}$

You can also draw out the portions multiply them separately, then fit them together to show your answer




The area of A is $1 \times 2 = 2$ square units.

The area of B is $1 \times \frac{1}{2} = \frac{1}{2}$ square unit

The area of C is $\frac{1}{2} \times 2 = 1$ square unit

The area of D is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ square unit

The total area is $2 + \frac{1}{2} + 1 + \frac{1}{4}$ square units, or $3\frac{3}{4}$ square units.

## 2.6 & 2.8 & 2.9 - Dividing fractions

- Dividing is seeing how many times you can split a number or fraction up. So if you look at 100 ÷ 5 = 20 you are actually seeing how many times 5 fits into 100, which is 20 times.
- This means if you are dividing fractions you can find your answer by using multiplication. take a look at this example:

$3 = \frac{3}{1}$

Dividing by 3 is the same as multiplying by $\frac{1}{3}$

$$\frac{9}{20} \div 3 = \frac{1}{3} \times \frac{9}{20} = \frac{1 \times 9}{3 \times 20} = \frac{9}{60} = \frac{9 \div 3}{60 \div 3} = \frac{3}{20}$$

So your steps for Dividing fractions are:

Step #1 - Convert terms to improper or proper fractions
Step #2 - Change the second fraction to it's reciprocal (flip the numerator and
r)
Step #3 - Multiply the numerators
Step #4 - Multiply the denominators
Step #5 - Put your answer into lowest terms or a mixed fraction

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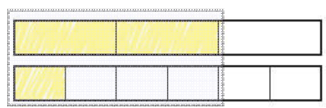
Take a look at it using fraction strips

Divide $\frac{2}{3}$ by 4.



$\frac{2}{3} \div 4 = \frac{1}{6}$

They took the portion that the fraction represented, split it into $\frac{1}{4}$ sections to show what product would be.

Calculate $\frac{1}{3} \div \frac{2}{5}$.

$$\frac{1}{3} \div \frac{2}{5} = \frac{1 \times 5}{3 \times 5} \div \frac{2 \times 3}{5 \times 3} = \frac{5}{15} \div \frac{6}{15} = \frac{5}{6}$$

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Take a look at these two different ways to divide!

$1\frac{7}{8} \div \frac{3}{5}$

$$1\frac{7}{8} \div \frac{3}{5} = \frac{15}{8} \div \frac{3}{5} = \frac{15 \times 5}{8 \times 5} \div \frac{3 \times 8}{5 \times 8} = \frac{75}{40} \div \frac{24}{40} = \frac{75}{24}, \text{ or } 3\frac{3}{24}, \text{ or } 3\frac{1}{8}$$

Here we find equivalent fractions with common denominators then divide the numerators and divide the denominators

Here we multiply by the reciprocal of the second fraction.

$$1\frac{7}{8} \div \frac{3}{5} = \frac{15}{8} \div \frac{3}{5} = \frac{15}{8} \times \frac{5}{3} = \frac{75}{24}, \text{ or } 3\frac{3}{24}, \text{ or } 3\frac{1}{8}$$

Both ways are correct!

<u>1.3, Adding and Subtraction Rational Numbers</u>

First off, before you add and subtract rational numbers convert them all to the same form, so you are adding or subtracting all fractions, all decimals or all integers.

**EXAMPLE 3** | Adding rationals in fraction form

Calculate $3\frac{1}{4} + \frac{-7}{3}$.

**Larissa's Solution**

$3\frac{1}{4} + \frac{-7}{3}$ is about $3 + (-2) = 1$. — I estimated first. $3\frac{1}{4}$ is a little more than 3 and $\frac{-7}{3} = -2\frac{1}{3}$ is about $-2$. I used $3 + (-2)$ to estimate.

$3\frac{1}{4} = \frac{13}{4} = \frac{39}{12}$

$\frac{-7}{3} = -\frac{7}{3} = -\frac{28}{12}$ — I wrote $3\frac{1}{4}$ as $\frac{13}{4}$. Then, I wrote each rational using equivalent fractions with the same denominator, 12.

$3\frac{1}{4} + \frac{-7}{3} = \frac{39}{12} + \left(-\frac{28}{12}\right) = \frac{11}{12}$ — I added the rationals by adding the numerators.

$\frac{11}{12}$ is close to 1, so my answer is reasonable. — I compared my result to my estimate to check.

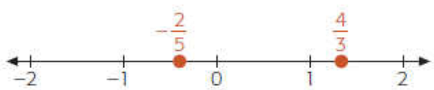
Negative fractions are not a big deal, they hold the same value as a negative decimal or whole number! Just think of them on a number line and be sure to have them placed correctly on the number line!

**EXAMPLE 4** | Subtracting rationals in fraction form

Calculate $1\frac{1}{3} - \frac{2}{-5}$.

**David's Solution: Using a number line to visualize**

$1\frac{1}{3} - \frac{2}{-5}$ is the distance from $\frac{2}{-5} = -\frac{2}{5}$ to $1\frac{1}{3} = \frac{4}{3}$ on a number line. — I know that to calculate a difference, I can think about the distance from the second value to the first one.




$\frac{2}{5} = \frac{6}{15}$ and $\frac{4}{3} = \frac{20}{15}$

$\frac{6}{15} + \frac{20}{15} = \frac{26}{15}$ or $1\frac{11}{15}$ — I added $\frac{2}{5}$ to get from $-\frac{2}{5}$ to 0 and $\frac{4}{3}$ to get from 0 to $\frac{4}{3}$.

Remember that subtraction is just the difference between two numbers, use a number line if you get confused

## In Summary

### Key Ideas

- Adding and subtracting rational numbers in the form of decimals combines the rules for adding and subtracting positive decimals with the rules for adding and subtracting integers.
  For example, $-4.3 + 5.25 = 5.25 - 4.3 = 0.95$.
- Adding and subtracting rational numbers in the form of fractions combines the rules for adding and subtracting positive fractions with the rules for adding and subtracting integers.
  For example, $5\frac{3}{4} - \left(-2\frac{1}{3}\right) = 5\frac{3}{4} + 2\frac{1}{3}$.

### Need to Know

- It is useful to estimate sums and differences to verify calculations of sums and differences.
- You can visualize a number line and use a combination of locations and distances to estimate and calculate sums and differences of rationals.

## 1.4, Multiplying and Dividing Rational Numbers

Remember, before you complete any operations with rational numbers convert them all to the same form, so you are adding, subtracting, multiplying or dividing all fractions, all decimals or all integers.

**In Summary**

**Key Ideas**

- Multiplying and dividing rational numbers in decimal form combines the rules for multiplying and dividing positive decimals with the rules for multiplying and dividing integers. For example,

$$(-3.2) \div 1.2 = -(3.2 \div 1.2)$$

- Multiplying and dividing rational numbers in the form of fractions combines the rules for multiplying and dividing positive fractions with the rules for multiplying and dividing integers. For example,

$$5\tfrac{3}{4} \times \left(-2\tfrac{1}{3}\right) = -\left(\tfrac{23}{4} \times \tfrac{7}{3}\right)$$

**Need to Know**

- You can divide rational numbers in the form of fractions by using a common denominator and dividing the numerators. For example,

$$-\frac{12}{25} \div \frac{3}{5} = -\frac{12}{25} \div \frac{15}{25}$$

- You can also divide by multiplying by the reciprocal. For example,

$$-\frac{12}{25} \div \frac{3}{5} = -\frac{12}{25} \times \frac{5}{3}$$

again be sure to convert rational numbers so that they are all in the same form before you complete any operations!

## The Sign Laws:

There are rules to follow whenever you are multiplying or dividing rational numbers.

If the signs are the same the answer will be positive
If the signs are the same the answer will be negative.

e.g.

| | |
|---|---|
| 5 * 5 = 25 | + multiplied by a + = + |
| -5 * -5 = 25 | - multiplied by a - = + |
| -5 * 5 = -25 | + multiplied by a - = - |
| 5 * -5 = -25 | - multiplied by a + = - |

## 1.5, Order of Operations with Rational Numbers

**In Summary**

**Key Idea**

- The rules for order of operations with rationals are the same as with integers:
  - Do what is in brackets first.
  - Multiply and divide from left to right.
  - Add and subtract from left to right.

## 2.10 - Order of Operations

- In order for math to work correctly there is a specific order that you have to complete the operations in (addition, subtraction, multiplication and division)

Use this to help you remember what order we complete math in!


B - Brackets
E - Exponents
D - Division
M - Multiplication
A - Addition
S - Subtraction

This order must always be followed when you need to complete more than one operation in a question.

(There is no exponents or division in this question)

Calculate $\frac{7}{3} - \frac{4}{5} \times \left(\frac{5}{6} \div \frac{1}{2}\right) + \frac{1}{4}$

$$\frac{7}{3} - \frac{4}{5} \times \left(\frac{5}{6} \div \frac{1}{2}\right) + \frac{1}{4}$$

$$= \frac{7}{3} - \frac{4}{5} \times \frac{10}{6} + \frac{1}{4}$$

$$= \frac{7}{3} - \left(\frac{4}{5} \times \frac{10}{6}\right) + \frac{1}{4}$$

$$= \frac{7}{3} - \frac{40}{30} + \frac{1}{4}$$

$$= \left(\frac{7}{3} - \frac{4}{3}\right) + \frac{1}{4}$$

$$= \frac{3}{3} + \frac{1}{4}$$

$$= 1\frac{1}{4}$$